Chapter 1

Subtraction

File: `arithmetic/sections/05_subtraction.ftl.tex`

[readtex `arithmetic/sections/04_ordering.ftl.tex`]

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Definition 1.1. Let n, m be natural numbers such that $n \geq m$. $n - m$ is the natural number k such that $n = m + k$.

Let the difference of n and m stand for $n - m$.

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Proposition 1.2. Let n, m be natural numbers such that $n \geq m$. Then

$$n - m = 0 \quad \text{iff} \quad n = m.$$

Proof. Case $n - m = 0$. Then $n = (n - m) + m = 0 + m = m$. End.

Case $n = m$. We have $(n - m) + m = n = m = 0 + m$. Hence $n - m = 0$. End. □

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Corollary 1.3. Let n be a natural number. Then

$$n - n = 0.$$

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Proposition 1.4. Let n be a natural number. Then

$$n - 0 = n.$$

Proof. We have $n = (n - 0) + 0 = n - 0$. □

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Proposition 1.5. Let n, m be natural numbers such that $n \geq m$. Then

$$n - m \leq n.$$

Proof. We have $(n - m) + m = n$. Hence $n - m \leq n$. □

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Proposition 1.6. Let n, m be natural numbers such that $n \geq m$. Then

$$0 \neq m < n \quad \text{implies} \quad n - m < n.$$

Proof. Assume $0 \neq m < n$. Suppose $n - m \geq n$. We have $(n - m) + m = n$. Then $n + m = (n - m) + m = n = n + 0$. Hence $m = 0$. Contradiction. □

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Proposition 1.7. Let n, m, k be natural numbers such that $n \geq m$. Then

$$(n - m) + k = (n + k) - m.$$

Proof. We have

$$\begin{aligned} &((n - m) + k) + m \\ &= ((n - m) + m) + k \\ &= n + k \end{aligned}$$

$$= ((n + k) - m) + m.$$

Hence $(n - m) + k = (n + k) - m$. □

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Corollary 1.8. Let n, m, k be natural numbers such that $n \geq m$. Then

$$k + (n - m) = (k + n) - m.$$

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Proposition 1.9. Let n, m, k be natural numbers such that $n \geq m + k$. Then

$$(n - m) - k = n - (m + k).$$

Proof. We have

$$\begin{aligned}
&((n - m) - k) + (m + k) \\
&= (((n - m) - k) + k) + m \\
&= (n - m) + m \\
&= n \\
&= (n - (m + k)) + (m + k).
\end{aligned}$$

Hence $(n - m) - k = n - (m + k)$. □